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# GUIDANCE RESEARCH

# TOWARDS REDUCED MISS DISTANCES

## *Final Technical Report*

covering the period of February 2001 – August 2002

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This Final Technical Report describes the continued validation of a new non-orthodox guidance law, compensating for the estimation delay due to noisy measurements. The guidance law was developed, based on a linearized planar interception model of constant velocities and fixed lateral acceleration bounds. The estimation process of noisy measurements was approximated by a delay in the estimated target acceleration. This delay was partially compensated by solving a deterministic delayed information pursuit-evasion game model.
The reported validation effort is an extension of earlier results included in an Interim Technical Report, submitted last year. It includes the summary of a great number of Monte Carlo simulations against two types of assumed target maneuvers. The results show that for each type of maneuver a different estimator/guidance law combination has to be used for obtaining the best homing accuracy.

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## Preface

The current research effort under AFOSR contracts No. F61775-01-WE018, funded by BMDO, is in fact a continuation of an earlier investigation performed under BMDO contract HQ00006-98-C-0014 during the period of September 1998 - August 1999. Due to administrative difficulties, the current effort could start only in February 2001. In January 2002 the contract was extended until August 2002.

The first part of this effort (February 2001 - September 2002) was summarized in an Interim Technical Report, submitted in November 2002, which covered the following tasks:

a) Reconfiguration of the 3D nonlinear simulation model of endo-atmospheric interception of maneuvering tactical ballistic missiles to include an on board estimator using noise corrupted measurements.

b) Validation of a new guidance law concept, compensating for the inherent information delay due to the estimation process in nonlinear 3D simulations of a realistic Theatre Missile Defense scenario.

c) Comparison of the linearized pursuit-evasion game model with bounded controls, used in the earlier investigation, to the classical linear quadratic game formulation.

This Final (Annual) Technical Report repeats (for sake of completeness) the problem formulation and the data base, but not the underlying theory, which was already published in the technical literature. The Final Technical Report concentrates on presenting and analyzing new results obtained by a very large set of Monte Carlo simulations in the context of extending the validation of the new guidance law.

# Abstract

This Final Technical Report describes the continued validation of a new non-orthodox guidance law, compensating for the estimation delay due to noisy measurements. The guidance law was developed, based on a linearized planar interception model of constant velocities and fixed lateral acceleration bounds. The estimation process of noisy measurements was approximated by a delay in the estimated target acceleration. This delay was partially compensated by solving a deterministic delayed information pursuit-evasion game model.

The reported validation effort is an extension of earlier results included in an Interim Technical Report, submitted last year. It includes the summary of a great number of Monte Carlo simulations against two types of assumed target maneuvers:

(i) a horizontal "bang-bang" maneuver randomly switched during the interception end-game;
(ii) a "spiral" maneuver generated by a fixed lateral asymmetry and non-zero trim angle of attack.

The results show that for each type of maneuver a different estimator/guidance law combination has to be used for obtaining the best homing accuracy.

# 1.  Introduction

The 1991 Gulf War reintroduced the threat of tactical ballistic missiles used as a terror weapon and presented a new challenge to the guided missile community. Successful interception of ballistic missiles requires a very small miss distance or even a direct hit. The new missile defense systems (such as PAC-3, Arrow, THAAD, etc.) have been designed against known tactical ballistic missiles and succeeded to demonstrate a "hit-to-kill" accuracy in the interception tests carried out with non-maneuvering targets. As a consequence of their successful development and eventual deployment, the threat of the currently operational tactical ballistic missiles will be minimized or even eliminated. However, the future threat of reentry vehicles with high maneuver potential is anticipated.

At the Faculty of Aerospace Engineering in the Technion, Israel Institute of Technology, a multi-year investigation has been performed (partially supported by AFOSR funds) concentrating in the analysis of future interception scenarios against highly maneuverable reentry vehicles, a problem beyond the horizon of the current missile defense projects. The first phase of this investigation, using simplified interception scenarios, indicated that classical guidance and estimation techniques are unable to achieve the required homing accuracy against such targets due to limited maneuverability advantage and the inherent estimation delay.

In an earlier investigation effort, sponsored by BMDO contract HQ00006-98-C-0014, a new non-orthodox interceptor guidance law compensating for the inherent information delay due to the estimation process of noisy measurements was developed. The derivation of this guidance law, denoted as DGL/C [1], was based on a simplified planar linearized model with constant velocities and fixed lateral acceleration bounds. The estimation process of the state variables required for the interception from noisy measurements was approximated by a delay in the estimated target acceleration. This delay was partially compensated by solving a deterministic delayed information pursuit-evasion game model [2]. Applying this new guidance law lead to a significant reduction of the guaranteed miss distance and restored the robustness with respect to the actual target maneuver. This approach represented a potential breakthrough in guidance law design and predicted reduced miss distances and robustness even in very stressing interception environments. The homing performance of the new guidance law was tested by a set of linearized Monte Carlo simulations, demonstrating the validity of the analytical predictions.

Due to administrative difficulties the research activity planned to extend the results of the planar constant speed model to a three-dimensional interception scenario with time varying speed, representing a realistic endo-atmospheric ballistic missile defense (BMD) scenario, could be continued only in February 2001, after awarding the present contract.   Fortunately, in a parallel effort under AFOSR contract F61708-97-C0004 a generalized time-varying linear pursuit-evasion game model was developed. Based on this model a modified guidance law with improved homing performance, denoted as DGL/E, was derived [3, 4]. Simulations of a kinematically realistic, but noise free, BMD scenario against a highly maneuvering generic reentry vehicle validated the predictions of the time-varying linear model.

The two separately developed improved guidance laws (DGL/C and DGL/E) were integrated to a new, still planar, guidance law, denoted as DGL/EC [5]. For application in a three-dimensional interception scenario this guidance law was implemented in two perpendicular channels of a generic interceptor missile. The implementation in a scenario with noise corrupted measurements was based on using a three-channels on-board Kalman Filter type estimator in seeker coordinates.

Testing the integrated estimator/guidance system in Monte Carlo simulation required reconfiguring the three-dimensional simulation model used in the earlier AFOSR effort. Including the three-channels on-board estimator and the measurement noise model in the simulation program was a substantial task and its functional verification has been a prerequisite for testing the homing performance of the new guidance law. This effort, together with the first phase of validation in a realistic nonlinear simulation model of a typical missile defense scenario, was described in the Interim Technical Report submitted in November 2001 [6].

This Final Technical Report repeats (for sake of completeness) the problem formulation and the data base, but not the underlying theory, which was already published in the technical literature and explained in detail in the Interim Technical Report. This Final report concentrates on presenting and analyzing new results obtained by a very large set of Monte Carlo simulations in the context of extending the validation of the new guidance law.

The structure of the report is the following. In the next section the formulation of a 3-D interception scenario of a maneuvering tactical ballistic missile (TBM) as a zero-sum pursuit-evasion game of imperfect information (the interceptor missile being the *pursuer* and the maneuvering TBM being the *evader*) is repeated. It is followed (section 3) by a summary of analytical results. Section 4 is devoted to simulation results. It includes a brief summary of previous results, the description of the simulation model and the data base used in the Monte Carlo simulations. The first part of the simulations was against a TBM performing random "bang-bang" type maneuvers. It is followed by simulation results against a TBM executing a typical "spiral" maneuver.

## 2. Problem formulation

2.1. Scenario description

The investigated interception scenario is the end game between an interceptor missile launched against a maneuverable reentering TBM. The scenario is characterized by a near head-on engagement of high velocity with variable altitude. For sake of simplicity a nominal point defense scenario trajectory is considered, *i.e.* the interceptor missile is launched from the vicinity of the TBM's target.

The initial position of the TBM is determined by assuming a non-maneuvering ballistic trajectory aimed at a fixed surface target. The initial position of the TBM also determines the vertical plane of reference. When the reentering TBM is detected, the defense system selects the desired altitude for interception and launches a guided missile towards the predicted point of impact at this altitude.

## 2.2. TBM model

The reentering TBM is assumed to be a generic cruciform flying vehicle having some control surfaces to execute lateral maneuvers up to a given angle of attack in fixed (non-rolling) body coordinates. The relationship between the actual angle of attack and its commanded value is approximated by a first-order transfer function. The generic TBM is characterized by its ballistic coefficient, which determines the deceleration in the atmosphere and the lift to drag ratio at the trimmed angle of attack generating lift. Due to some eventual asymmetry, the reentering TBM also rotates (rolls) around its longitudinal axis and (having a non-zero trim angle of attack) follows a "spiral" type trajectory.

## 2.3. Interceptor model

The generic interceptor missile (designed by a group of students for high endo-atmospheric interception) has an aerodynamically controlled cruciform airframe and is assumed to be roll stabilized. It has solid rocket propulsion of two stages. Each rocket motor stage provides a constant thrust. After the "burn out" of the first stage the booster is separated and the second rocket motor is ignited. The ignition of the second propulsion stage can be delayed to allow maximum interceptor velocity in the end game, which starts when the onboard seeker "locks on" the target. The maneuverability of the missile (its lateral acceleration and the corresponding load factor) is limited, in each of the two perpendicular planes of the cruciform configuration, by the maximum lift coefficient. It is assumed that the missile's autopilot can be represented by a first-order transfer function.

## 2.4. Simplifying assumptions

In the analysis of the investigated interception end-game scenario (with a high closing velocity) the following simplifying assumptions were made:

(A-1) The relative end-game trajectory can be linearized around a fixed reference line such as the initial line of sight.

(A-2) The velocity profiles of both missiles on a nominal trajectory are known and can be expressed as the function of time.

(A-3) The maximum lateral acceleration of each missile is known as a function of time.

(A-4) Both missiles can be represented by point-mass models with linear control dynamics.

(A-5) The maneuvering dynamics of both missiles can be approximated by first order transfer functions.

(A-6) The target has no information on the state of the interceptor.

(A-7) The interceptor has noisy measurements of the target relative position.

## 2.5. Lethality model

A realistic lethality model between an interceptor's warhead and its target, depending on many physical parameters, is very complex. In this point-mass study the probability of destroying the target is determined by the following simplified lethality function.

$$P_d = \begin{cases} 1 & M \le R_k \\ 0 & M > R_k \end{cases} \tag{2.1}$$

where $R_k$ is the lethal (kill) radius of the warhead and M is the miss distance. This model assumes an overall reliability of 1 of the entire guidance system.

## 2.6. Performance index

The interception end game is formulated as a zero sum imperfect information pursuit-evasion game, where the objective of the interceptor (*pursuer*) is to destroy the incoming target (*evader*) with a predetermined probability of success, with the smallest possible lethal radius $R_k$ of the warhead. The required probability of success is assumed to be 0.95 against all feasible target maneuvers. This probability is denoted as the single shot kill probability (SSKP) defined by

$$SSKP = E\{P_d(R_k)\} \tag{2.2}$$

where $E$ is the mathematical expectation taken over the entire set of noise samples against any given feasible target maneuver. Using this definition the cost function of this game, to be minimized by the *pursuer* and maximized by the *evader*, is

$$J = R_k = \arg\{SSKP = 0.95\} \tag{2.3}$$

## **3. Summary of analytical results**

Based on the above outlined assumptions and formulation several zero-sum pursuit-evasion games were solved. The solution of each such game included three elements: (i) the optimal guidance law (*optimal pursuer strategy*); (ii) the worst target maneuver (*optimal evader strategy*); (iii) the guaranteed miss distance (*the value of the game*).

### 3.1. DGL/1

The first model that was solved was a *perfect information* game, where assumptions (A-6) and (A-7) were replaced by assuming that all the state variables and the game parameters are known to both *players*. Moreover, in this game planar interception geometry, as well as constant velocities and maneuverabilities were considered. Based on (A-1) and (A-2) the final time of the interception can be computed for any given initial conditions of the end game, allowing to define the time-to-go ($t_{go}=t_f-t$), which becomes the independent variable of the problem. The set of assumptions (A-1) – (A-5) allows casting the problem to the canonical form of linear games, from which a reduced order game can be obtained [7]. For a planar

interception geometry there is only a single state variable, the *zero effort miss distance,* well known in missile guidance analysis and denoted as Z. The solution of such a game is determined by two parameters of physical significance: the *pursuer/evader* maneuverability ratio ($\mu = a_p^{max}/a_e^{max}$) and the *evader/pursuer* time constant ratio ($\varepsilon = \tau_e/\tau_p$). The solution results in the decomposition of the reduced game space ($t_{go}$, Z) into two regions of different strategies, as it can be seen in Fig. 1. These regions are separated by the pair of optimal boundary trajectories denoted respectively by $Z^*_+$ and $Z^*_-$, reaching tangentially the Z=0 axis at $(t_{go})_s$. where $(t_{go})_s$ is the non zero root of the equation $dZ/dt_{go} = 0$.
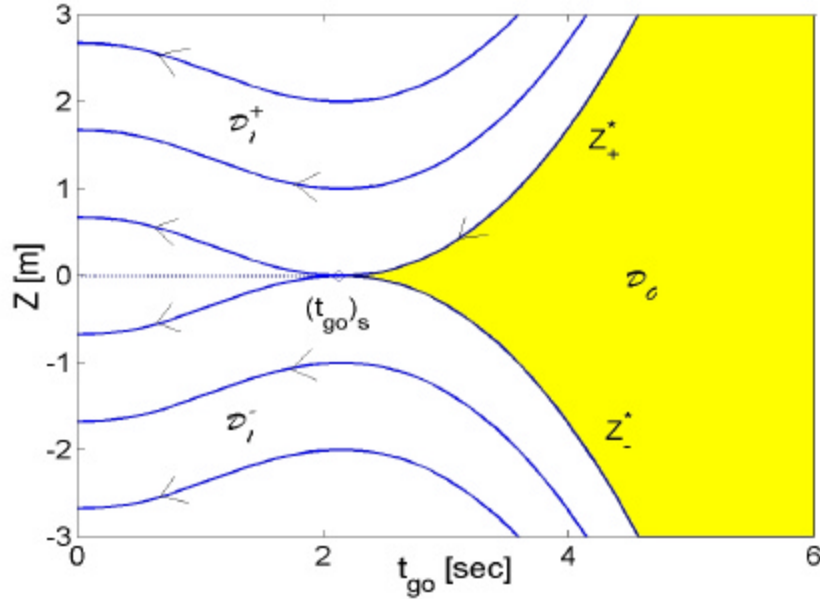


Fig. 1. Example of game space decomposition

One of the regions is a *regular* one, denoted by $\boldsymbol{D_1}$, where the optimal strategies of the players are of the "bang-bang" type

$$\mathbf{u}^* = \mathbf{v}^* = \text{sign } \{Z\} \quad \forall \quad Z \neq 0 \tag{3.1}$$

**u** and **v** being the normalized controls of the *pursuer* (interceptor) and the *evader* (the maneuvering target) respectively. The *value* of the game in this region is a unique function of the initial conditions. The boundary trajectories themselves also belong to $\boldsymbol{D_1}$. Inside the other region, denoted by $\boldsymbol{D_0}$, the optimal strategies are arbitrary and the *value* of the game is constant, depending on the parameters of the game ($\mu, \varepsilon$). If the parameters of the game are such that $\mu\varepsilon \geq 1$, then the only root of $dZ/dt_{go} = 0$ is zero, and the *value* of the game in $\boldsymbol{D_0}$ is also zero. As a consequence $\boldsymbol{D_0}$, which includes all initial conditions of practical importance, becomes the *capture zone* of the game. Note that the "bang-bang" strategies (3.1) are also optimal in $\boldsymbol{D_0}$.

The practical interpretation of this game solution is the following: (i) the optimal missile guidance law can be selected as (3.1) for the entire end game; (ii) the worst target maneuver is a constant lateral acceleration starting not after $(t_{go})_s$; (iii) the guaranteed miss distance depends on the parameters ($\mu, \varepsilon$) and can be made zero if $\mu\varepsilon \geq 1$. Implementation of the optimal missile guidance law, denoted as DGL/1,

requires the perfect knowledge of the *zero effort miss distance*, which includes also the current lateral acceleration of the target.

## 3.2. DGL/E

The second model is also a planar *perfect information* game, but with time varying velocities and maneuverabilities. Such a model is much more suitable to the analysis of a realistic ballistic missile defense scenario. The solution of this game [4], although it is qualitatively very similar to the previous one, depends strongly on the respective velocity and maneuverability profiles of the *players*. Obviously the value of $\mu$ is not constant. Nevertheless, in some cases the condition that "the only root of $dZ/dt_{go} = 0$ is zero" can be satisfied and in these cases the game has a non empty *capture zone*. Due to the time varying velocity and maneuverability profiles, the expressions of the *zero effort miss distance*, as well as of $(t_{go})_s$ and the guaranteed miss distance, become more complex.

In spite of this (algebraic) complexity, the implementation of the optimal missile guidance law, denoted as DGL/E, doesn't present essential difficulties compared to the implementation of DGL/1. It requires, of course, in addition of the perfect knowledge of the current lateral acceleration of the target, the velocity and maneuverability profiles in the end game, as indicated by the assumptions (A-2) and (A-3). The interceptor profiles can be directly measured on board, while the velocity and maneuverability profiles of the target can be precalculated along a nominal trajectory.

## 3.3. DGL/C

As mentioned earlier, the implementation of the perfect information guidance laws DGL/1 and DGL/E require the knowledge of the target lateral acceleration. Since this variable cannot be directly measured, this variable has to be estimated based on noise corrupted measurements. Analyzing the estimation performance by extensive simulation studies with different types of estimators, noise models and random target maneuver structures, it was found that the greatest error source in the interception scenario of maneuvering targets is the inherent delay in estimating time varying target maneuvers. Based on this observation, a rough approximation of the estimation process assumed that the evader's lateral acceleration is a perfect outcome that is delayed by $\Delta t_{est}$, while the estimation of the other state variables is ideal. This modeling assumption allowed a deterministic analysis.

If the *pursuer* uses DGL/1, derived from the perfect information game solution [7] the *evader* can take advantage of the estimation delay and achieve a large miss distance by adequate optimal maneuvering [8], even if the game parameters are such that the guaranteed miss distance should be zero. Therefore, a new *pursuit-evasion game* had to be formulated and solved. The solution of this planar *"delayed information"* game [2], assuming for sake of simplicity constant velocities and lateral acceleration bounds, was based on the idea of *reachable sets* suggested for such problems [8]. Based on this approach the *zero effort miss distance* Z is replaced by the center of the *uncertainty set* created by the estimation delay, denoted ac $Z_c$. The decomposition of the reduced game space $(t_{go}, Z_c)$ seems qualitatively similar to Fig. 1. However, due to the delay the guaranteed miss distance cannot be zero and it is a

monotonically increasing function of the estimation delay divided by the time constant of the *evader* ($\delta = \Delta t_{est}/\tau_e$). The solution yielded a guidance law, denoted as DGL/C, which partially compensates for the inherent estimation delay [1]. The deterministic analysis guarantees that using this guidance law a substantial reduction of the guaranteed miss distance and a robust guidance performance with respect to the target maneuver structure can be achieved.

The main difficulty in implementing DGL/C is to determine the value of $\Delta t_{est}$ to be used in the expression of $Z_c$. This value depends both on the structure and parameters of the estimator, as well as on the measurement noise model. For a given estimator and noise, the "best" value of $\Delta t_{est}$ that minimizes the miss distance distribution against the "worst" target maneuver has to be found by a min-max search using off-line Monte Carlo simulations.

3.4. DGL/EC

The two different improvement features, developed using planar linearized interception models (DGL/E with perfect information and time varying velocities, DGL/C with constant velocities and delayed information), were integrated into a single (still planar) guidance law denoted as DGL/EC [5]. This planar guidance law was implemented in two perpendicular guidance channels of a roll-stabilized inteceptor missile and tested in simulations of a generic but realistic noise corrupted nonlinear ballistic missile defense scenario by using a suitable three-dimensional estimator [8]. The estimation process was carried out in the rotating sensor frame. The measurements were range, azimuth and elevation in the sensor frame and. the angular velocity of the sensor frame itself. The resulting three-dimensional estimator consisted of three independent filters, whose states included position, velocity and acceleration along each sensor axis. The rotation of the sensor frame was considered using the instantaneously frozen coordinate system approach.

The first simulation experiment of an interception end game using DGL/EC, carried out against the "worst" (horizontal "bang-bang" type) target maneuver, was reported in [5] and it confirmed the predictions of the deterministic analysis.

## 4. Simulation results ("bang-bang" target maneuver)

4.1 Previous results

The simulation program used in [5] was later adapted for an extensive validation and sensitivity study. The first phase of the adaptation was reported in [6]. The modular 3-D nonlinear point-mass simulation program was developed for ballistic missile defense scenarios against highly maneuvering TBMs. The original program, described in [6], included the following elements: 3-D nonlinear relative kinematics between two point-mass vehicles, point-mass dynamics and first order control dynamics of each flying vehicle, a 3-D estimator with a measurement frequency of 200 Hz and a high-altitude standard atmospheric model. The simulations were carried out in a fixed Cartesian coordinate system, assuming flat non-rotating earth and no wind using the well-known equations of 3-D kinematics. For each test point 100 simulations with different measurement noise samples were used. The measurement noise was assumed to be zero mean Gaussian with the same constant

angular standard deviation in elevation and azimuth.

The simulations reported in [6] confirmed the main findings of [5], namely the substantial reduction of the miss distances achieved by using DGL/EC compared to DGL/1 against the "worst" target maneuver (a horizontal "bang–bang" type maneuver with the "worst" reversal timing). The results of [6] reaffirmed that in spite of the impressive improvement of homing performance hit-to-kill homing accuracy against highly maneuvering agile targets cannot yet be achieved. The estimator used in [5] and [6] was a Kalman filter with an exponentially correlated acceleration (ECA) shaping filter, suggested by Singer, having a first order time constant. The simulation results of [6] indicated that the value of this time constant affects the estimation delay. However, if the delay is compensated by using DGL/EC, the warhead's lethal radius that guarantees SSKP of 0.95, denoted as $R_{k.95}$, is almost insensitive to the value of this time constant. Nevertheless, for larger noise levels a larger value of the time constant seems to yield slightly better results.

## 4.2. New simulation objectives

Based on the results reported in [6], the simulation tasks for completing the objectives of the present contract were determined as follows.

a) A basic sensitivity study with respect to target parameters;

b) A sensitivity study with respect to estimator parameters;

c) A comparison of two different estimators (shaping filters);

d) Investigation of the interception of a TBM performing "spiral" maneuvers.

Note that the first three items were based on simulating interception of a TBM performing a horizontal "bang-bang" type maneuver, as in earlier studies. The last topic addresses the interception of a very different target and therefore it is presented in a separate section.

Moreover, in order to allow flexibility in changing the initial conditions of the interception end game, the existing modular simulation program was augmented with a subprogram for computing the launch conditions against a given incoming target to be intercepted at a desired altitude.

## 4.3. Computation of launch conditions

The subprogram performing this task assumed that the target is detected at the altitude of 150 km and is aimed to hit a surface asset collocated with the interceptor launch site. The velocity and the flight path angle of the incoming target were calculated based on an assumed initial range of 600 km and an optimal initial launch angle. Based on these assumptions at the altitude of 150 km (210.3 km of horizontal separation from the intended surface target collocated with the launch site) the velocity of the target was 1720 m/sec and its flight path angle was $-18.2^0$. From these "initial" conditions a nominal (non maneuvering) target trajectory was computed using a ballistic coefficient of $\beta = 5000 \text{kg/m}^2$.

In order to intercept the "nominal" target at the desired altitude the subprogram calculates the time of launching and the initial flight path angle of the interceptor. It is required that the interceptor velocity at the nominal interception altitude be maximal (for best maneuverability). This requirement can be achieved by selecting the appropriate delay for the ignition of the second stage rocket motor. Using this subprogram the simulation of the interception end game becomes more realistic.

The interceptor is launched according to the precomputed results of the subprogram and guided from the ground to reach the "nominal" interception point until the "lock-on" range of the interceptor's seeker (20 km in the examples used in this report) is reached. During that time the target trajectory is simulated, allowing also eventual maneuvers with a lift/drag ratio of $\Lambda = 2.6$. Due to the differences between the precomputed ("nominal") and the simulated ("real") trajectories, the initial conditions of the interception end game are not "ideal" and the end game stars with some initial error as in reality.

4.4. Simulation data base

All simulated interception end games, reported here, we aimed at a "nominal" interception altitude of 20 km and assumed a 3 second delay between the ignition of the second stage rocket motor after "burnout" of the first stage. The first stage operates 6.5 sec and provides a trust of 229 kN. The second stages operates 13 sec and provides a trust of 103 kN. The specific impulse of both rocket motors is 250 sec. The initial mass of the interceptor is 1540 kg. The maneuverability of the interceptor (its lateral acceleration and the corresponding load factor) is limited, in each of the two perpendicular planes of the cruciform configuration, by the maximum lift coefficient. It is assumed that the interceptor's autopilot can be represented by a first-order transfer function with a time constant $\tau_p = 0.2$ sec.

By using the interceptor cross section as reference surface ($S_{ref} = 0.2$ m$^2$) the maximum lift coefficient is $C_{Lmax} = 1.8$. The zero lift drag coefficient of the interceptor (rocket motor on) is $C_{D0} = 0.25$. Based on the above data the "nominal" velocity and maneuverability profiles of the interceptor and the target during the end game were computed and are shown in Figs. 2 and 3. In Fig. 4 the time varying interceptor/target maneuverability ratio is plotted. It can be seen that during the end game this parameter that characterizes the interceptor advantage is monotonically (almost linearly) decreasing. The entire end game can be characterize by the final value of this parameter, denoted as $\mu_f$.
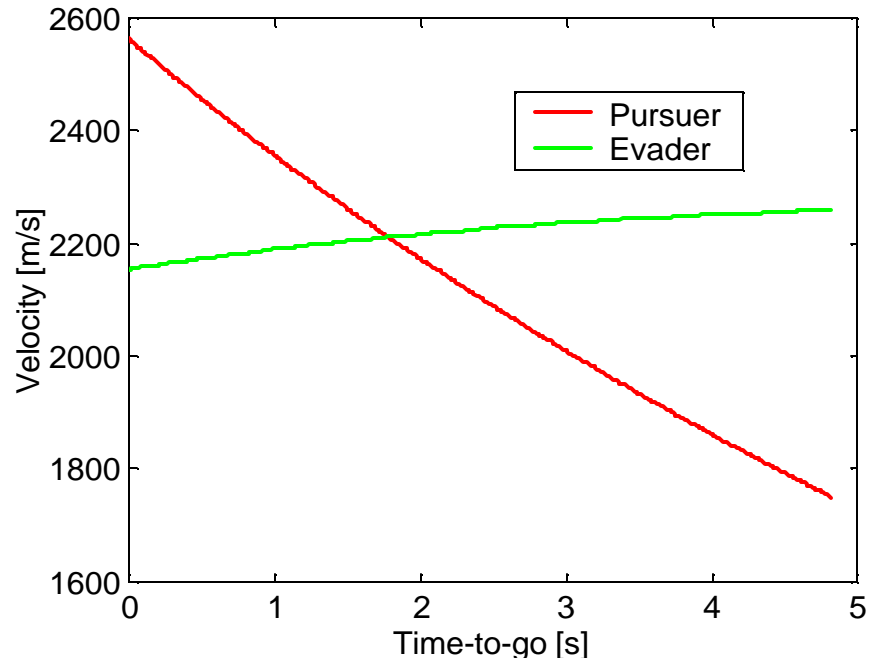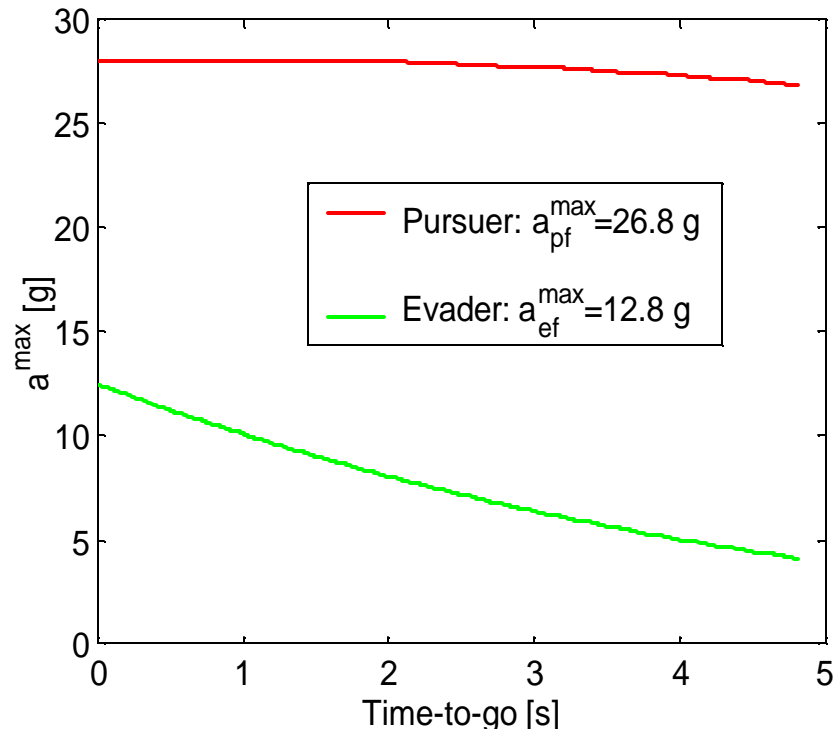
Fig. 2. Nominal velocity profiles



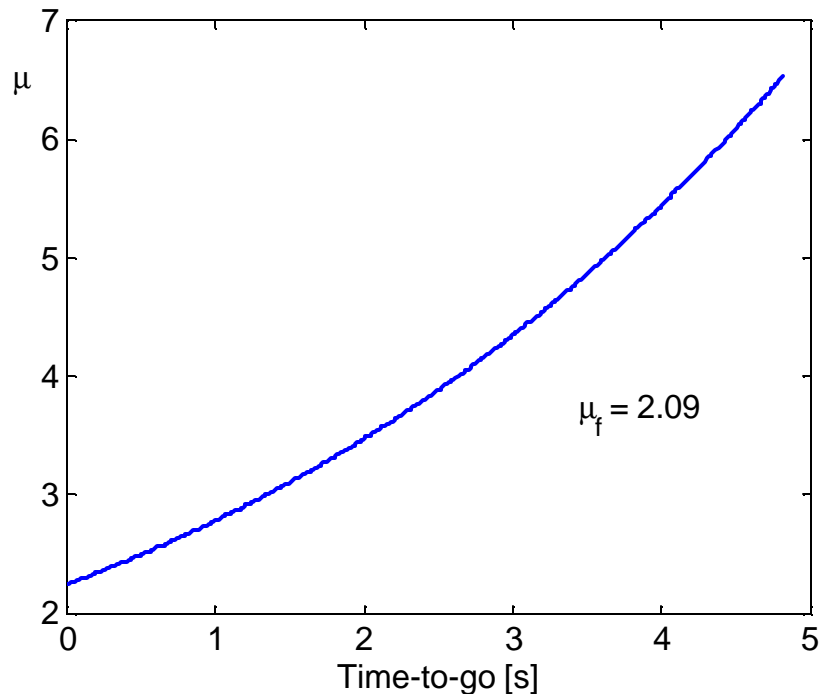Fig. 3. Nominal maneuverability profiles

Fig. 4. Nominal maneuverability ratio

4.5. Sensitivity to target parameters

A large set of Monte Carlo simulations were carried out with different values of the target parameters ($\Lambda$, $\tau_e$) and angular noise levels ($\sigma_{az} = \sigma_{el}$) against the "worst" target maneuver using the same estimator and the guidance law DGL/C with the "best" delay compensation. These simulations yielded predictable results. For fixed interceptor parameters (including the estimator) and noise levels, the value of $R_{k.95}$, used as the measure of homing performance, is monotonically increasing with target maneuverability, expressed by $\Lambda$, and decreasing with increasing $\tau_e$. Similarly, for constant target and interceptor parameters, $R_{k.95}$ is a monotonic function of the noise level. Since in reality target parameters are not well known, both pessimistic and optimistic guesses can be made. The difference in these guesses is illustrated in Fig. 5, where the miss distance distributions of two cases are shown. The comparison was made between a "nominal" pessimistic guess ($\Lambda = 2.6$, $\tau_e = 0.2$ sec and $\sigma_{az} = \sigma_{el} = 0.1$ mrad) and an optimistic one assuming ($\Lambda = 1.3$, $\tau_e = 0.4$ sec, $\sigma_{az} = \sigma_{el} = 0.05$ mrad). While the optimistic case predicts a hit-to-kill accuracy, the pessimistic guess requires a warhead with a lethal radius of at least 2.5m for an SSKP of 0.95.
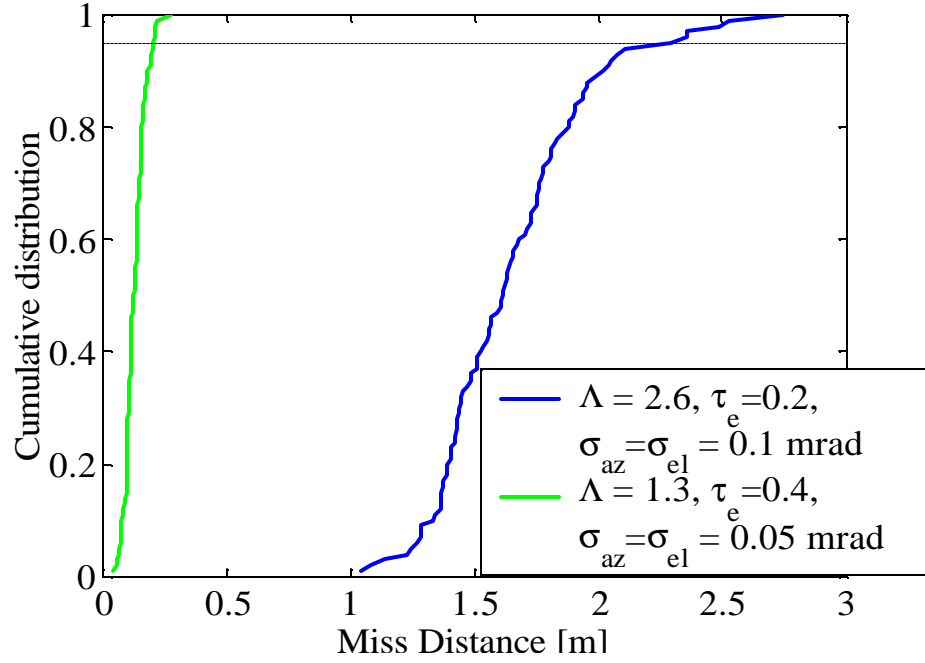
Fig. 5. Miss distance distributions for different assumptions

### 4.6. Sensitivity to estimator parameters

As mentioned earlier the estimator used in [5] and [6] was a Kalman filter with ECA shaping filter. This is a first order filter driven by with a zero mean white noise representing the random target maneuver model [9]. The spectral density of the noise, supposed to be proportional to the maneuvering energy of the target, is expressed by

$$Q_{ECA} = C_{ECA} \, (a_e^{max})^2 \qquad\qquad (4.1)$$

Such an estimator has two tuning parameters: the time constant $\tau_a$, which supposed to be inversely proportional to the assumed average frequency of target maneuver switches, and the proportionally factor $C_{ECA}$ of the noise spectral density. In [6] it was found that the estimation delay of such an estimator is proportional to the value of $\tau_a$. Moreover, such an estimator provides biased estimates of the relative lateral velocity and the lateral target acceleration. The magnitudes of the estimated values of these variables both variables are smaller than the actual ones. The biases are inversely is proportional to the value of $\tau_a$.

In order to represent random "bang-bang" type maneuvers in [9] another type of shaping filter, called random starting time (RST) filter is suggested. This is an integrator driven by zero mean white noise. The spectral density of the noise is proportional to the square of the maximum target lateral acceleration and inversely proportional to the duration of the end game

$$Q_{RST} = C_{RST} \, (a_e^{max})^2 / t_f \qquad\qquad (4.2)$$

where the factor of proportionality (denoted by $C_{RST}$) can serve as the tuning parameter of the estimator. This type of estimator turns out to be unbiased and its delay in estimating the relative lateral velocity and the lateral target acceleration is proportional to the value of the tuning parameter $C_{RST}$.

As the first phase of the sensitivity study, the homing performance of the interceptor, implementing the uncompensated guidance law DGL/E against a nominal target ($\Lambda = 2.6$, $\tau_e = 0.2$ sec) performing the "worst" maneuver switch and a measurement noise level of $\sigma_{az} = \sigma_{el} = 0.1$ mrad, was compared using both estimators with different tuning parameters. The values of $R_{k.95}$ of this comparison are summarized in Table 1.

Table 1.  Worst-case $R_{k,95}$ using DGL/E [m]

| $\tau_s$, s \ C | 1 | 2 | 3 |
|---|---|---|---|
| 0.5 | 1.19 | 1.29 | 1.50 |
| 1.0 | 1.42 | 1.58 | 1.78 |
| 1.5 | 1.53 | 1.63 | 1.89 |
| RST | 1.71 | 1.87 | 2.17 |

In order to present a unified approach the spectral density of the noise in both shaping filters are expressed by

$$Q = (a_e^{max}/C)^2 \tag{4.3}$$

where for ECA estimator

$$C = \sqrt{1/C_{ECA}} \tag{4.4}$$

and for RST estimator

$$C = \sqrt{t_f/C_{ECA}} \tag{4.5}$$

One can see that with the DGL/E the estimator with ECA shaping filter leads to better results than with RST. The results also indicate that the value of $R_{k.95}$ is monotonically increasing with the tuning parameters.

In the next phase the "best" values of the estimation delays to be compensated were found and applied using DGL/EC. The values of these "optimal" delays are presented in Table 2, while the resulting values of $R_{k.95}$ are summarized in Table 3. Although the estimation delay values are quite small, their compensation by DGL/EC leads to a meaningful homing improvement.

| Table 2 . "Optimal" delays, [sec] | | | | Table 3.  Worst-case $R_{k,95}$ using DGL/EC [m] | | | |

Table 2 . "Optimal" delays, [sec]

| $\diagdown$ C<br>$\tau_s$, s $\diagdown$ | 1 | 2 | 3 |
|---|---|---|---|
| 0.5 | - | 0.05 | 0.08 |
| 1.0 | 0.05 | 0.09 | 0.11 |
| 1.5 | 0.06 | 0.11 | 0.07 |
| RST | 0.1 | 0.15 | 0.07 |

Table 3.  Worst-case $R_{k,95}$ using DGL/EC [m]

| $\diagdown$ C<br>$\tau_s$, s $\diagdown$ | 1 | 2 | 3 |
|---|---|---|---|
| 0.5 | 1.19 | 1.11 | 1.28 |
| 1.0 | 1.14 | 1.11 | 1.34 |
| 1.5 | 1.11 | 1.11 | 1.52 |
| RST | 1.21 | 1.11 | 1.68 |

The results in Table 3 reveal several interesting phenomena. For both estimators there exists an optimal value of the noise spectral density leading to similar homing performance with DGL/EC. For the ECA shaping filter tuned to this spectral density value, the homing performance is insensitive to the value of filter's time constant, reconfirming the results already obtained in [6]. In Fig. 6 the miss distance distributions with two different shaping filters, the best RST and one of the best ECA are compared, showing a slight advantage to the ECA shaping filter.
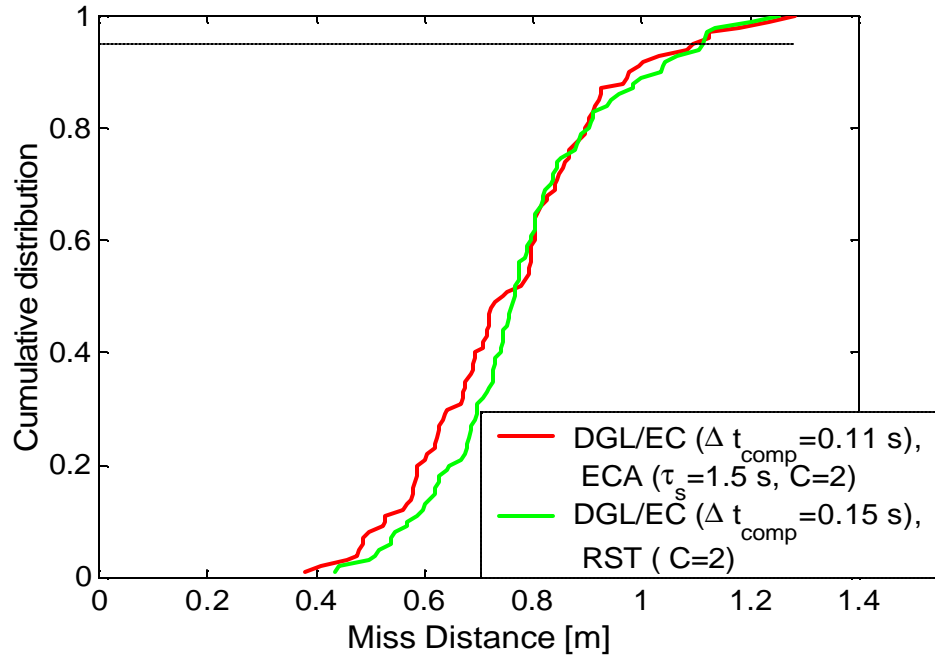


Fig. 6. Comparison of miss distance distributions using DGL/EC

# 5. Interception of a target with a "spiral" maneuver

In many studies the reentering TBM is assumed to perform a spiral maneuver. Although this type of maneuver is not the "optimal" evasion in the deterministic sense, it is still a rather efficient one. This type of maneuver is created if the reentering TBM has some lateral asymmetry together with a non zero trim angle of attack in a fixed direction of the body coordinates The lateral asymmetry creates a rolling moment proportional to the ambient dynamic pressure, which also creates a lift force in a constant body direction. Due to aerodynamic damping the roll rate, denoted by "p", is governed by a first order linear differential equation with time varying coefficients

$$\dot{p} + b\rho V p = c\rho V^2 \qquad (5.1)$$

where "b" and "c" are constant parameters proportional to the roll damping coefficient of the airframe and to the lateral asymmetry respectively. In the examples presented in this report the value of "b" was kept constant (b = 0.004 $m^2$/kg) and the value of "c" varied. For each value of "c" a different roll rate profile is obtained as it can be seen in Figs. 7 and 8.
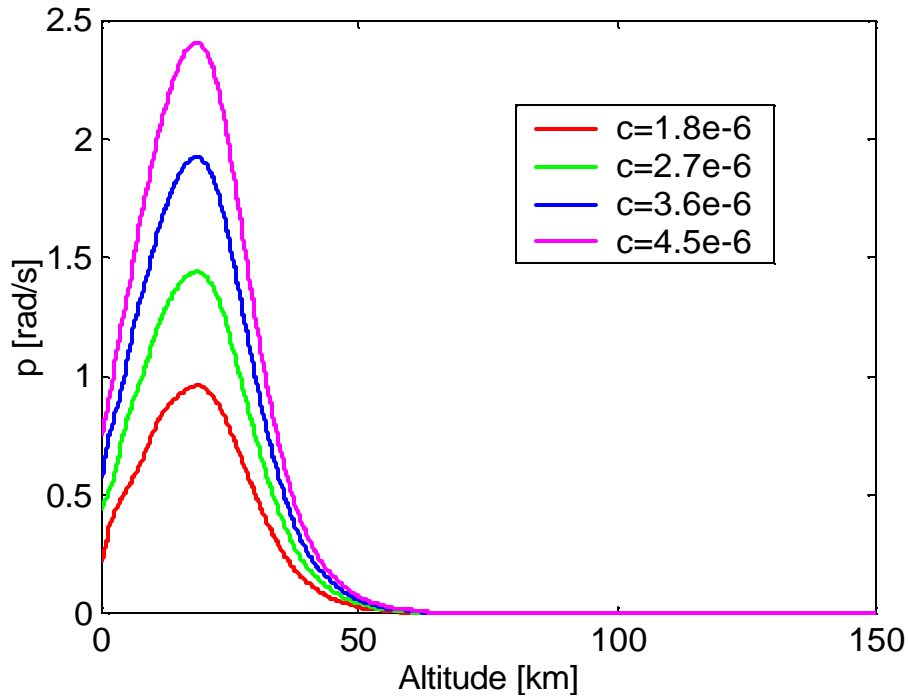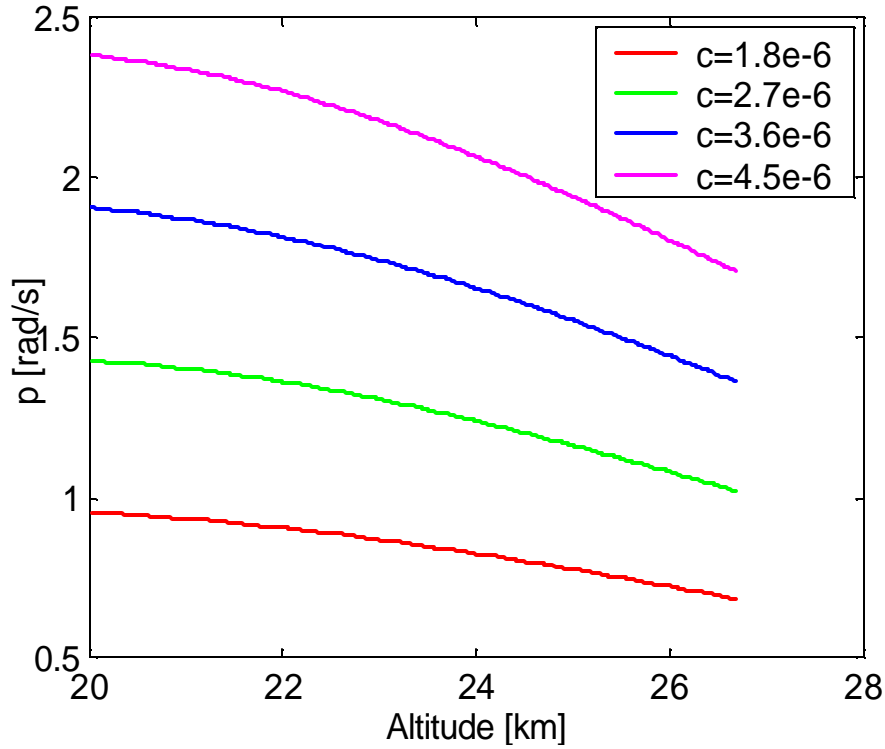


Fig. 7. Roll rate profiles

Fig. 8. End game roll rate profiles

The actual numerical value of "c" is of little significance. The important feature is the (average) roll rate and the value of the lateral acceleration (in target body coordinates), which generate a rotating acceleration vector normal to the target velocity. Any "spiral" trajectory can be generated from a given set of initial condition. In the simulations it was assumed that at the altitude of 150 km the roll rate is zero but the direction of the non zero trim angle of attack with respect to the horizon (i.e. the initial value of the roll angle) is arbitrary. With different initial roll angles different "spiral" TBM trajectories were generated and against each trajectory interception end games were simulated.

The results revealed a great sensitivity of the homing performance to the initial roll angle. Therefore, similarly to the interceptions against "bang-bang" type maneuvers, the "worst" case was considered. It was also found that for both for very low and very high roll rates the miss distances are small. Applying DGL/EC it was observed that the largest miss distances were created by an average roll rate of the order of 2 rad/sec, as it can be seen on Fig. 9.
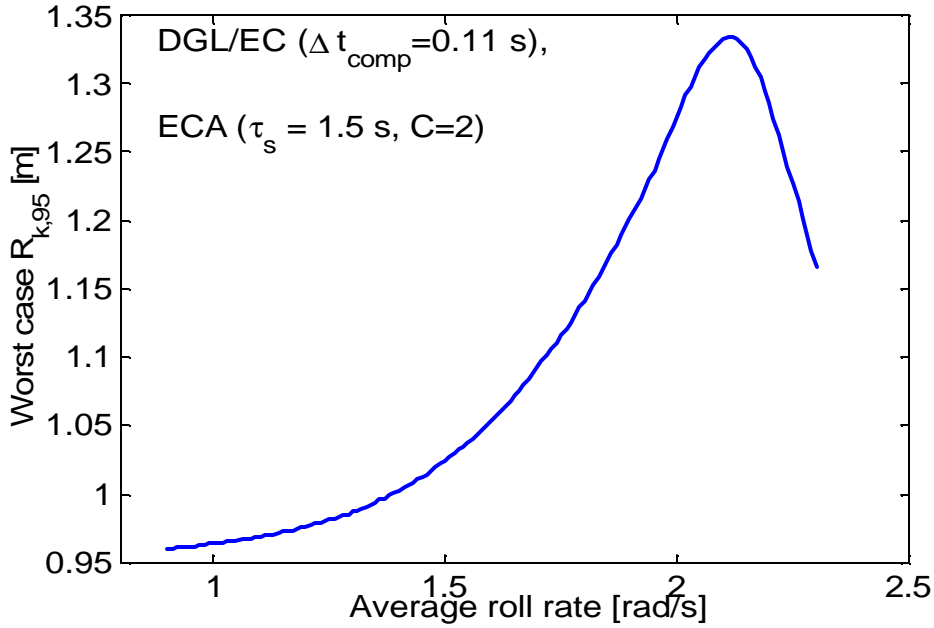
Fig. 9. Worst case miss distance vs. average end game roll rate

Interestingly, there was no significant difference whether ECA or RST shaping filters (tuned earlier for best results against the worst "bang-bang" maneuver switch) were incorporated in the estimator. It was also observed, that quite surprisingly, the homing performance against the worst "spiral" maneuver was inferior to the homing performance against the worst "bang-bang" maneuver as it is shown in Fig. 10.
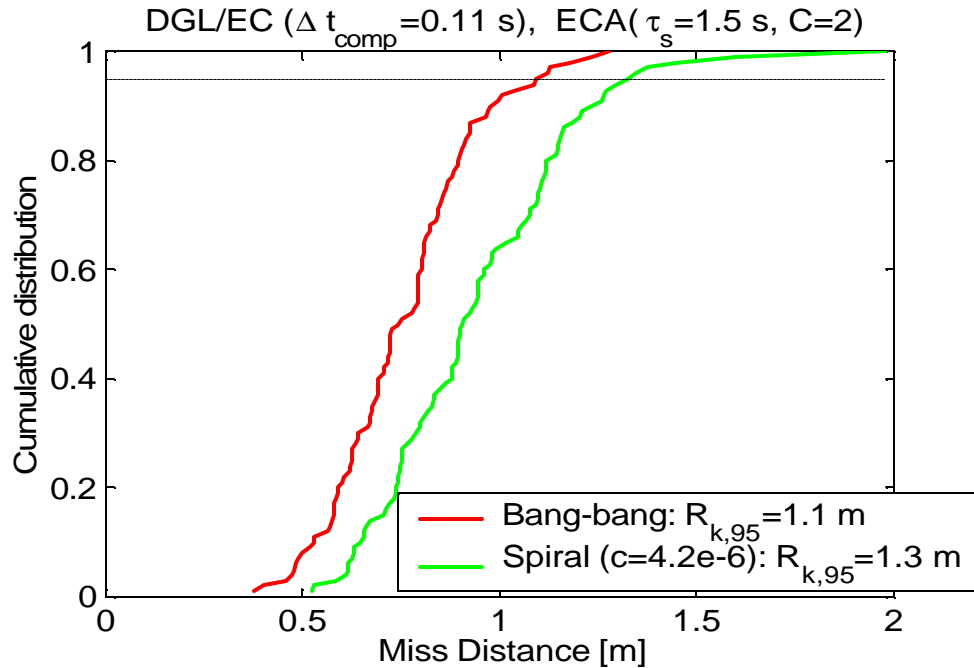


Fig 10. Comparison of worst-case miss distance distributions, same estimator.

Thorough investigation of the results indicated that the origin of the poor homing performance against a "spiral" maneuver are very large estimation errors, due to the unsuitability of the ECA and RST shaping filters to represent the quasiperiodical (random phase) maneuver. To achieve improved homing performance a new estimator with a periodical shaping filter [8], which is denoted as PSF, was designed. The assessment of the estimation results also leads to conclude that the "pure delay" estimation model, used in the development of DGL/C is not valid against periodical maneuvers. For this reason a new set of Monte Carlo simulations were initiated using DGL/E with the new estimator, denoted as PSF (periodical shaping filter). With this estimator the homing performance turned out to be much improved as it can be seen in Fig. 11. Tuning the shaping filter to the exact actual ("worst") roll rate has little effect on the estimation and homing performance.
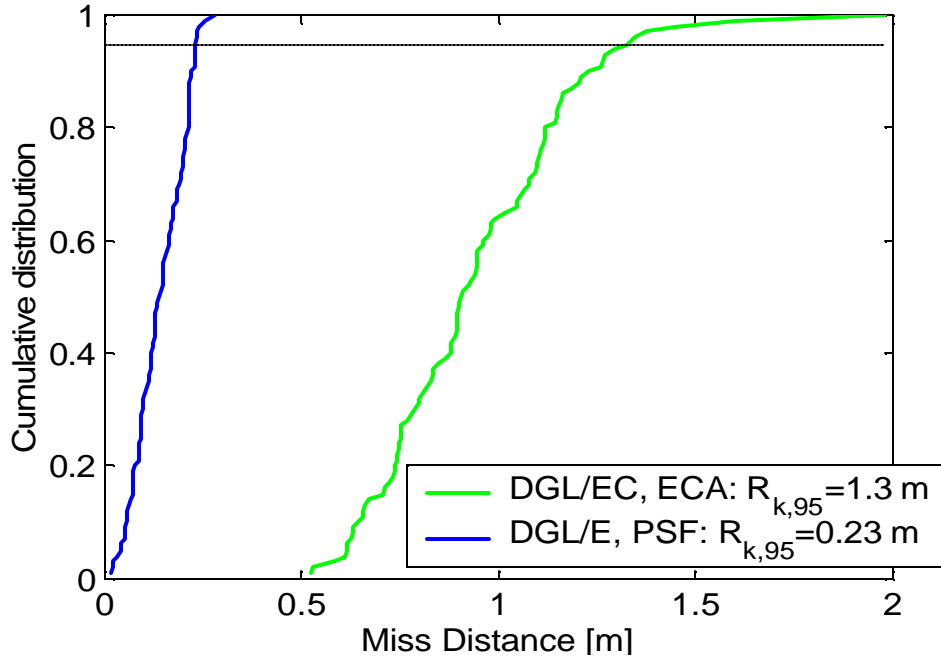


Fig. 11. Worst case miss distances against "spiral" maneuver, different estimators

The homing performance of DGL/E with the new PSF estimator against a "bang-bang" type maneuver, shown in Fig 12, was very poor, as it could be expected. This result reaffirms the assertion that the worst "spiral" maneuver is much less efficient than the worst "bang-bang" type maneuver with the same lateral acceleration as it can be seen in Fig.13. The results clearly indicate that an estimator/guidance law combination, which produces the best results against a given type of evasive maneuver, is not the best against a different maneuver type.

Although perfect information game theory states that applying the optimal *pursuer* strategy the *value* of the game is guaranteed against any *evader* strategy, such robustness is lost in realistic situations where the measurements are noise corrupted and estimators are required to implement the guidance law based on the optimal *pursuer* strategy against unknown (not necessarily optimal) evasive maneuvers.
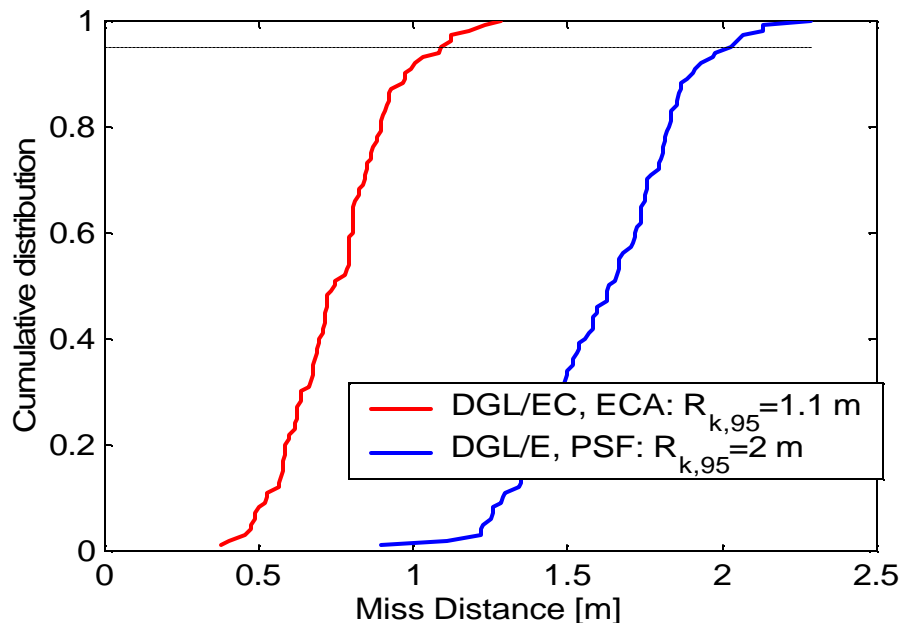
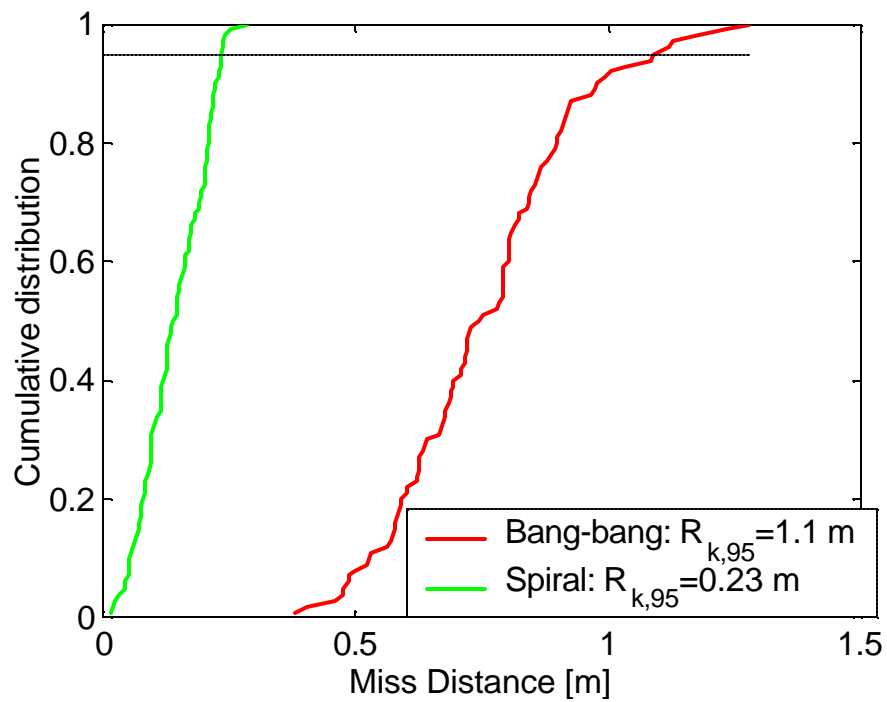Fig. 12. Worst case miss distances against "bang-bang" maneuver, different estimators



Fig.13. Comparison of "worst" maneuvers using optimized estimator/guidance law

# 6. Conclusions

This Final Technical Report describes the second part of the research conducted under AFOSR contract No. F61775-01-WE018. It is based on the results of a very large number of Monte Carlo simulations. For the sake of conciseness, only the main results are presented and discussed, but those reflect the compilation of huge bulk of "raw" data. The augmentation of the basic simulation code with the subprogram that computes the launch conditions and the timing for interception at a desired altitude provided an additional flexibility to an exhaustive sensitivity study.

The most important lesson learned from this large amount of simulation can be phrased as follows.

***In investigating the interception of a highly maneuverable TBM, the estimation and the guidance problems cannot be separately treated.***

Even if the type of target maneuver is well defined, the estimator parameters affect the guidance law and for each guidance law there is a different "optimal" estimator of a given structure.

The results with the "spiral" maneuver clearly demonstrate that there is no unique "optimal" estimator/guidance law combination against different types of target maneuvers. Finding the satisfactory solution against all feasible target maneuvers is a still unsolved research problem.

The results of the reported investigation provide an enhanced insight into the complex problem of intercepting highly maneuverable targets and emphasize the importance of further research towards a satisfactory solution for the critical task of future ballistic missile defense.

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